A. Introduction

-- In this chapter, we will study several ways of finding solutions of pair of linear equations.

B. Pair of Linear Equations in Two variables

-- An equation which can be put in the form $ax + by + c = 0$, where *a, b* and *c* are real numbers, and *a* and *b* are not both zero, is called a linear equation in two variables *x* and *y*. The solution of such an equation is a pair of values, one for *x* and the other for *y,* which makes the two sides of the equation equal.

-- Geometrically, it means every solution of the equation is a point on line representing it, each solution (x, y) of a linear equation in two variables, $ax + by + c = 0$, corresponds to a point on the line representing the equation.

-- The general form for a pair of linear equations in two variables *x* and *y* is:

$a_1x + b_1y + c_1 = 0$, and
$a_2x + b_2y + c_2 = 0$
where $a_1, b_1, c_1, a_2, b_2, c_2$ are all real numbers and
$a_1^2 + b_1^2 \neq 0,\ a_2^2 + b_2^2 \neq 0,$

-- Consider two lines in a plane, only one of the following three possibilities can happen:

(i) The two lines will intersect at one point.
(ii) The two lines will not intersect, i.e., they are parallel.
(iii) The two lines will be coincident.

A. Graphic Method of Solution of a Pair of Linear Equations

The common point on both the lines, is the solution for a pair of linear equations in two variables.

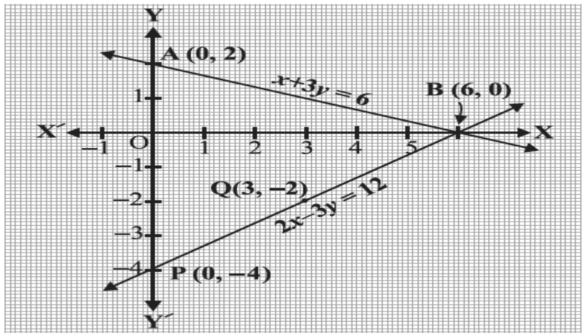
-- A pair of linear equations which has no solution, is called an *inconsistent*. A pair of linear equations which has a solution, is called a *consistent*. A pair of linear equations which has infinitely many distinct common solutions, is called a *dependent* (*consistent*) *pair of solutions.*

Sl No.	Pair of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratios	Graphical representation	Algebraic interpretation
1.	$x - 2y = 0$ $3x + 4y - 20 = 0$	$\frac{1}{3}$	$\frac{-2}{4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (unique)
2.	$2x + 3y - 9 = 0$ $4x + 6y - 18 = 0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
3.	$x + 2y - 4 = 0$ $2x + 4y - 12 = 0$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{-4}{-12}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

-- Two lines Intersect at one point:

$x + 3y = 6.$ $\qquad$ $2x – 3y = 12$

x	0	6	x	0	3
y	2	0	y	-4	-2

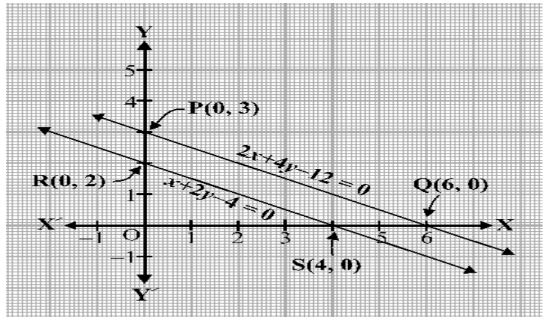


The solution of the pair of linear equations is x = 6 and y = 0, i.e., the given pair of equations is consistent.

-- Two lines are Parallel:

$x + 2y -4 = 0.$ $\qquad$ $2x + 4y – 12 = 0$

x	0	4	x	0	6
y	2	0	y	3	0

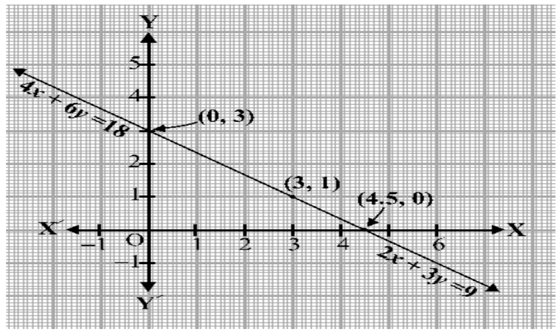


The parallel lines have no common point and thus no solution, the pair of equations is inconsistent.

-- Two lines are Coincident:

$2x + 3y =9.$ $\qquad$ $4x + 6y = 18$

x	0	4.5	x	0	3
y	3	0	y	3	1



The parallel lines have infinite many solutions, the pair of equations is dependent (consistent).