Class-IX AREAS OF PARALLELOGRAMS AND TRIANGLES

A. Introduction

-- It may be recalled that the part of the plane enclosed by a simple closed figure is called a *planar region*. The magnitude or measure of this planar region is called its *area*. It is always expressed with the help of a number (in some unit) such as 5 cm^2, 8 m^2 etc. So, the area of a figure is a number associated with the part of the plane enclosed by the figure.

-- We are also familiar with the concept of congruent figures. *Two figures are called congruent, if they have the same shape and the same size*.

-- So *if two figures A and B are congruent, they must have equal areas*. However, *two figures having equal areas need not be congruent*.

-- *The area of a figure is having following properties:*

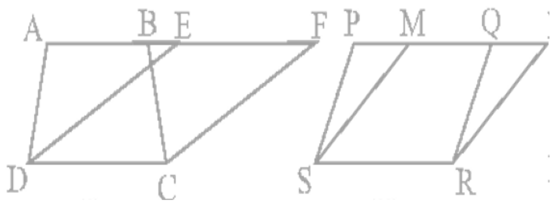
(1) *If* A *and* B *are two congruent figures, then*
$ar(A) = ar(B)$; *and*

(2) *If a planar region formed by a figure* T *is made up of two non-overlapping planar regions formed by figures* P *and* Q, *then*
$ar(T) = ar(P) + ar(Q)$.

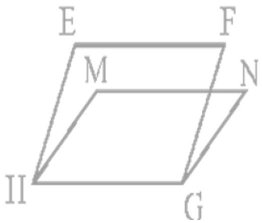
-- In this chapter, we study some relationship between the areas of these geometric figures under the condition when they lie on the same base and between the same parallels.

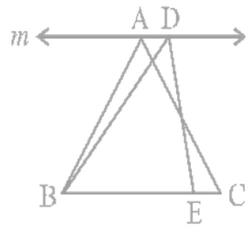
B. Figures on the Same Base and Between the Same Parallels

Two figures are said to be on the same base and between the same parallels, if they have a common base (side) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.



The trapezium ABCD and parallelogram EFCD are on the *same base* DC, the vertices A and B (of trapezium ABCD) and E and F (of parallelogram EFCD) opposite to base DC lie on a line AF parallel to DC. Thus, are on *the same base* DC *and between the same parallels* AF and DC. Similarly, parallelograms PQRS and MNRS are on the same base SR and between the same parallels PN and SR.

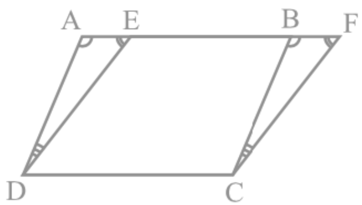




The parallelograms EFGH and MNGH do not lie between the same parallels EF and HG. The ΔABC and ΔDBE are not on the common base. So, it should clearly be noted that *out of the two parallels, one must be the line containing the common base.*

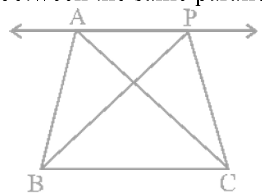
C. Parallelograms on the same Base and Between the Same Parallels

Theorem : *Parallelograms on the same base (or equal bases) and between the same parallels are equal in area.*



D. Triangles on the Same Base and between the Same Parallel

Let us have two triangles ABC and PBC on the same base BC and between the same parallels BC and AP.



Theorem : Two *triangles on the same base* (*or equal bases*) *and between the same parallels are equal in area*.

-- *The area of a triangle is half the product of its base* (*or any side*) *and the corresponding altitude* (*or height*). From this formula, you can see that *two triangles with same base* (*or equal bases*) *and equal areas will have equal corresponding altitudes.*

Theorem : Two *triangles having the same base* (*or equal bases*) *and equal areas lie between the same parallels*.

-- It may also be noted that:
The median of a triangle divides it into two triangles of equal areas. Let ABC be a triangle and let AD be one of its medians, then,
$ar(ABD) = ar(ACD)$