# Class-XII RELATION AND FUNCTIONS

## A. Introduction

-- The concept of the term 'relation' in mathematics has been drawn from the meaning of relation in English language, according to which two objects or quantities are related if there is a recognisable connection or link between the two objects or quantities. Let A be the set of students of Class XII of a school and B be the set of students of Class XI of the same school. Examples:

(i) $\{(a, b) \in A \times B : a$ is brother of $b\}$,

(ii) $\{(a, b) \in A \times B : a$ is sister of $b\}$.

## B. Types of Relations

-- A relation R in a set A is called *empty relation*, if no element of A is related to any element of A, i.e.,

$R = \varphi \subset A \times A$.

-- A relation R in a set A is called *universal relation*, if each element of A is related to every element of A, i.e.,

$R = A \times A$.

-- The empty set $\emptyset$ and $A \times A$ are two extreme relations. Both the empty relation and the universal relation are also called *trivial relations*.

-- A relation R in a set A is said to be an *equivalence relation* if R is reflexive, symmetric and transitive.

-- A relation R in a set A is called

(i) *reflexive*, if $(a, a) \in R$, for every $a \in A$,

(ii) *symmetric*, if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

(iii) *transitive*, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, for all $a_1, a_2, a_3 \in A$.

-- *Equivalence class* $[a]$ containing $a \in X$ for an equivalence relation R in X is the subset of X containing all elements $b$ related to $a$. The equivalence relation partitions the set A into mutually exclusive equivalence classes.

A = A1 + A2 + A3 + A4 .. An

Subsets A1, A2, A3,… An are Equivalence classes.

## C. Type of Functions

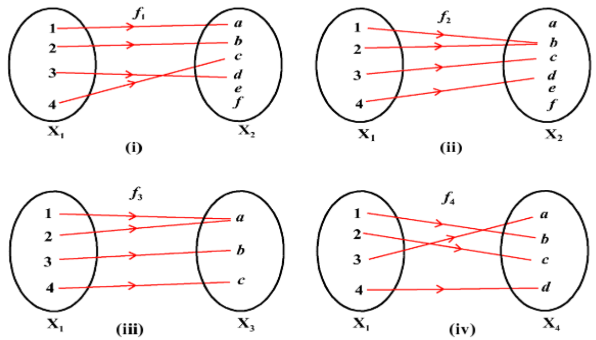
-- A function $f : X \to Y$ is defined to be *one-one* (or *injective*), if images of distinct elements of X under $f$ are distinct, i.e., for every $x_1, x_2 \in X$, $f(x_1) = f(x_2)$ implies $x_1 = x_2$. E.g (i) & (iv)

Otherwise, $f$ is called *many-one*. E.g. (ii) & (iii)

-- A function $f : X \to Y$ is said to be *onto* (or *surjective*), if every element of Y is the image of some element of X under $f$, i.e., for every $y \in Y$, there exists an element $x$ in X such that $f(x) = y$. (iii) & (iv)

$f$: $X \to Y$ is onto if and only if Range of $f = Y$.

-- A function $f : X \to Y$ is said to be *one-one* and *onto* (or *bijective*), if $f$ is both one-one and onto. E.g. (iv)




## D. Composition of Functions & Invertible Function

-- Let $f : A \to B$ and $g : B \to C$ be two functions. Then the *composition of f and g*, denoted by $gof$, is defined as the function $gof : A \to C$ given by $gof\,(x) = g(f(x))$, $\forall x \in A$.

It can be verified in general that $gof$ is one-one implies that $f$ is one-one. Also, $gof$ is onto implies that $g$ is onto.

-- A function $f : X \to Y$ is defined to be *invertible*, if there exists a function $g$: $Y \to X$ such that $gof = I_X$ and $fog = I_Y$. The function $g$ is called *inverse of f* and is denoted by $f^{-1}$. Thus, if $f$ is invertible, then $f$ must be one-one and onto and conversely, if $f$ is one-one and onto, then $f$ must be invertible.

Theorem: If $f : X \to Y$, $g : Y \to Z$ and $h : Z \to S$ are functions, then $ho\,(gof) = (hog)\, o\, f$.

Theorem: Let $f$: $X \to Y$ and $g$: $Y \to Z$ be two invertible functions. Then $gof$ is also invertible with $(gof)^{-1} = f^{-1}og^{-1}$.

## E. Binary Operations

-- It is to be noted that when we need to add three numbers, we first add two numbers and the result is then added to the third number. Thus, addition, multiplication, subtraction and division are examples of binary operation, as 'binary' means two. If we want to have a general definition which can cover all these four operations, then the set of numbers is to be replaced by an arbitrary set X and binary operation is nothing but association of any pair of elements $a$, $b$ from X to another element of X.

-- *A binary operation* $*$ on a set A is a function $* : A \times A \to A$. We denote $*(a, b)$ by $a * b$.

-- A binary operation $*$ on the set X is called *commutative*, if $a * b = b * a$, for every $a, b \in X$.

-- A binary operation $* : A \times A \to A$ is said to be *associative* if $(a * b) * c = a * (b * c)$, $\forall\, a, b, c, \in A$.

-- Given a binary operation $* : A \times A \to A$, an element $e \in A$, if it exists, is called *identity* for the operation $*$, if $a * e = a = e * a$, $\forall\, a \in A$.

-- Given a binary operation $* : A \times A \to A$ with the identity element $e$ in A, an element $a \in A$ is said to be *invertible* with respect to the operation $*$, if there exists an element $b$ in A such that $a * b = e = b * a$. The element $b$ is called the *inverse of a* and is denoted by $a^{-1}$.

-- Zero is identity for the addition operation on **R** but it is not identity for the addition operation on **N**, as $0 \notin$ **N**. In fact the addition operation on **N** does not have any identity.

-- Also for the addition operation $+ : \mathbf{R} \times \mathbf{R} \to \mathbf{R}$, given any $a \in \mathbf{R}$, there exists $-a$ in **R** such that $a + (-a) = 0$ (identity for '+') $= (-a) + a$.

-- Similarly, for multiplication operation on **R**, given any $a \neq 0$ in **R**, we can choose $\frac{1}{a}$ in **R** (identity for '×') $= \frac{1}{a} \times a$.