Class-X POLYNOMIALS

A. Introduction

Polynomial: An algebraic expression with non-negative integral power of variables. e.g. $p(x) = 2x^3 + 3x + 5$. It contains one or more Algebraic terms. $1/x + 5$, $x^{-2} + x - 7$, are not polynomials.
-- The highest power of variable in polynomial is called degree of polynomial.

Classification based on Degree of Variables:

1. Linear e.g. $p(x) = 4x + 5$.
2. Quadratic e.g. $p(x) = 2x^2 + x + 3$.
3. Cubic e.g. $p(x) = 2x^3 + 3x + 5$.

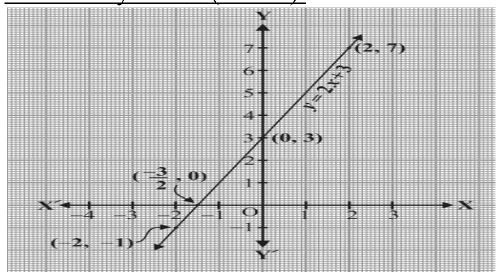
Value of Polynomial: If p(x) is a polynomial in x, and if k is any real number, then the value obtained by replacing x by k in p(x), is called the value of p(x) at x = k, and is denoted by p(k).

Zeroes of a Polynomial: For p(x), there are all the x-values that make the polynomial equal to zero. These are also called roots of a polynomial.
-- The zeroes of polynomial are calculated by equating it to zero, (p(x) = 0) and solving to get the values of x.

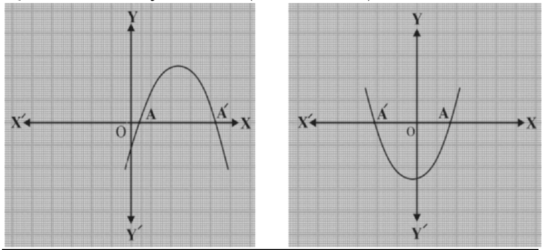
B. Geometrical Meaning of Zeroes of a Polynomial

The zero of a polynomial is the x-coordinate of the point where the graph intersects the x-axis.

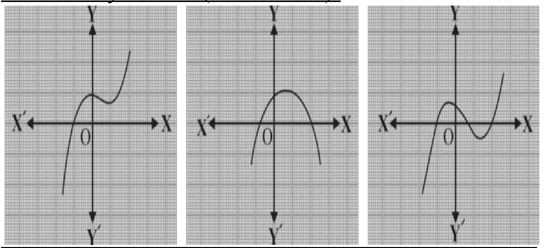
Linear Polynomial (1 Zero):



Quadratic Polynomial (0-2 Zeroes):



Cubic Polynomial (0-3 Zeroes):



C. Relationship between Zeroes and Coefficients of a Polynomial

Linear Polynimial: In general, if k is a zero of $p(x) = ax + b$, then $p(k) = ak + b = 0$, i.e., $k = \frac{-b}{a}$.
So, the zero of the linear polynomial $ax + b$ is $= \frac{-(constant\ term)}{Coefficient\ of\ x}$

Quadratic Polynomial: In general, if α and β are the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, $a \neq 0$, then you know that $x - \alpha$ and $x - \beta$ are the factors of $p(x)$. Therefore,
$ax2 + bx + c = k(x - \alpha)(x - \beta)$, where k is a constant
$= kx^2 - k(\alpha + \beta)x + k\alpha\beta$.

Comparing the coefficients of x^2, x and constant terms on both the sides, we get
$a = k$, $b = -k(\alpha + \beta)$ and $c = k\alpha\beta$.

This gives,
sum of zeroes $= \alpha + \beta = \frac{-b}{a} = \frac{-(Coefficient\ of\ x)}{Coefficient\ of\ x2}$
product of zeroes $= \alpha\beta = \frac{c}{a} = \frac{Constant\ term}{Coefficient\ of\ x2}$

Cubic Polynomial: In general, if α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, then

$\alpha + \beta + \gamma = \frac{-b}{a}$,
$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$,
$\alpha\beta\gamma = \frac{-d}{a}$.

D. Division Alogrithm for Polynomials

-- A cubic polynomial has at most three zeroes. However, if one zero is given, we can find the other two.
Dividend = Divisor x Quotient + Remainder

Division Alogrithm: If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that
$p(x) = g(x) \times q(x) + r(x)$
where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.
This result is known as the Division Algorithm for polynomials.

Example: Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, *if you know that two of its zeroes are* $\sqrt{2}$ *and* $-\sqrt{2}$.
-- Since two zeroes are $\sqrt{2}$ and $-\sqrt{2}$, $(x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$ is a factor of the given polynomial. Now, we divide the given polynomial by $x^2 - 2$.
So, $2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x^2 - 2)(2x^2 - 3x + 1)$.
Solving thr quadratic polynomial by factorisation, we get the zeroes of the given polynomial as:
$\sqrt{2}, -\sqrt{2}, \frac{1}{2}$, and 1.