Class-XII MATRICES

A. Matrices

Matrix: A *matrix* is an ordered rectangular array of *real numbers or functions taking real value*, called the *elements or the entries* of the matrix. The horizontal lines of elements are said to constitute, *rows* of the matrix and the vertical lines of elements are said to constitute, *columns* of the matrix.
Order of Matrix: A matrix having m rows and n columns is called a *matrix of order* $m \times n$.
--The number of elements in $m \times n$ matrix will be mn.

B. Types of Matrices

Column Matrix: A matrix is said to be a *column matrix* if it has only one column. In general, $A = [a_{ij}]_{m \times 1}$ is a column matrix of order $m \times 1$.
Row Matrix: A matrix is said to be a *row matrix* if it has only one row. In general, $A = [a_{ij}]_{1 \times n}$ is a row matrix of order $1 \times n$.
Square matrix: A matrix in which the number of rows are equal to the number of columns, is said to be a *square matrix*. In general, $A = [a_{ij}]_{m \times m}$ is a square matrix of order m.
Diagonal Matrix: A square matrix $A = [a_{ij}]_{m \times m}$ is said to be a *diagonal matrix* if all its non-diagonal elements are zero.
Scalar Matrix: A diagonal matrix is said to be a *scalar matrix* if its diagonal elements are equal,
Identity Matrix: A diagonal matrix is said to be a *identity matrix* if its diagonal elements are equal to 1.
Zero Matrix: A matrix is said to be *zero matrix* or *null matrix* if all its elements are zero.

C. Equality of matrices

Two matrices A and B] are said equal if ,
(i) they are of the same order, and
(ii) each element of A is equal to the corresponding element of B. Symbolically A = B.

D. Operations on Matrices

Addition of Matrices:
--If A and B are two matrices of the same order, then A+B i.e. the sum is a matrix obtained by adding the corresponding elements of the given matrices.
--If A and B are not of same order, A+B is not defined.
Negative of a Matrix: The negative of a matrix is denoted by – A. We define $-A = (-1) A$.
Difference of Matrices: If A and B are two matrices of the same order, then A-B i.e. the difference is a matrix obtained by subtracting the corresponding elements of the given matrices.
--In other words, $C = A - B = A + (-1) B$, that is sum of the matrix A and the matrix – B.
Properties of Matrix Addition:
- Commutative Law $A + B = B + A$.
- Associative Law $(A + B) + C = A + (B + C)$.
- Additive identity: Zero O is additive identity.
- Additive inverse: (–) A is additive inverse of A.

Multiplication by a scalar: if A is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each element of A by scalar k.
Properties of Scalar Multiplication:
- $k(A+B) = kA + kB$,
- $(k + l) A = kA + lA$

Multiplication of Matrices: The *product* of two matrices A and B is *defined* only if the number of columns of A is equal to the number of rows of B. Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Then the product of the matrices A and B is the matrix C of order $m \times p$. To get the element of the matrix C, we take the i^{th} row of A and k^{th} column of B, multiply them elementwise and take the sum of all these products.
--If AB is defined, then BA need not be defined.
--If AB and BA are both defined, it is not necessary that AB = BA. Multiplication of diagonal matrices of same order will be commutative.
--Zero matrix as the product of two non-zero matrices.

Properties of Matrix Multiplication:
Associative law: $(AB) C = A (BC)$.
Distributive law: $A(B+C)=AB+AC$, $(A+B) C = AC + BC$.
Multiplicative identity: Identity matrix, $IA = AI = A$.

E. Transpose of a Matrix

--If A be $m \times n$ matrix, then matrix obtained by interchanging the rows and columns of A is called the *transpose* of A. It is denoted by A' or (A^T).
Properties of Transpose of Matrices;
- $(A')' = A$,
- $(kA)' = kA'$ (where k is any constant)
- $(A + B)' = A' + B'$
- $(AB)' = B'A'$

F. Symmetric and Skew Symmetric Matrices

--A square matrix A is said to be *symmetric* if $A' = A$ and is said to be *skew symmetric* matrix if $A' = -A$.
Theorems:
-For any square matrix A with real number entries, $A + A'$ is symmetric matrix and $A - A'$ is skew symmetric matrix.
-Any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.

G. Elementary Operations (Transformation)

- The interchange of any two rows or two columns.
- The multiplication of the elements of any row or column by a non-zero number.
- The addition to the elements of any row or column, the corresponding elements of any other row or column multiplied by any non-zero number.

H. Invertible Matrices

--If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that $AB = BA = I$, then B is called the *inverse* matrix of A and it is denoted by A^{-1}. In that case A is said to be invertible.
--Inverse of a square matrix, if it exists, is unique.
--If A and B are invertible matrices of the same order, then $(AB)^{-1} = B^{-1} A^{-1}$.