Class-XII DIFFERENTIAL EQUATIONS

A. Introduction

-- This chapter deals with study of some basic concepts related to differential equation, general and particular solutions of a differential equation, formation of differential equations, some methods to solve a first order first degree differential equation and some applications of differential equations in different areas.

B. Basic Concepts

Differential Equations: An equation involving derivative (derivatives) of the dependent variable with respect to independent variable (variables) is called a *differential equation*, e.g.

$$x\frac{dy}{dx} + y = 0$$

Ordinary Differential Equation: It is the differential equation involving derivatives of the dependent variable with respect to only one independent variable, e.g.

$$2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

Partial Differential Equation: It is the differential equations involving derivatives with respect to more than one independent variables,

Order of a differential equation: It is the order of the highest order derivative of the dependent variable with respect to the independent variable involved in the given differential equation, e.g.

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + y = 0 \quad \text{(Order is 3)}$$

Degree of a differential equation: It is defined when it is a polynomial equation in derivatives. It is the highest power (positive integral index) of highest order derivative, e.g.

$x\frac{dy}{dx} + y = 0$, $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$. (In both cases, degree is one)

-- Order and degree (if defined) of a differential equation are always positive integers.

C. General and Particular Solutions of Differential Equations

-- The solution of the differential equation is a function φ that will satisfy it i.e., when the function φ is substituted for the unknown y (dependent variable) in the given differential equation, L.H.S. becomes equal to R.H.S.

A. Verify that the function $y = e^{-3x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

$y = e^{-3x}$

$\frac{dy}{dx} = -3e^{-3x}$

$\frac{d^2y}{dx^2} = 9e^{-3x}$

Substituting the values in given differential equation, we

L.H.S.$=9e^{-3x} + (-3e^{-3x}) - 6.e^{-3x} = 9e^{-3x} - 9e^{-3x} = 0 =$ R.H.S..

-- The solution which contains arbitrary constants is called the *general solution* (*primitive*) of the differential equation. The solution free from arbitrary constants i.e., the solution obtained from the general solution by giving particular values to the arbitrary constants is called a *particular solution* of the differential equation.

D. Formation of Differential Equations whose General Solution is Given

Procedure to form differential equations that will represent a given family of curves

-- To form a differential equation from a given function we differentiate the function successively as many times as the number of arbitrary constants in the given function and then eliminate the arbitrary constants.

(a) If the given family F_1 of curves depends on only one parameter *a,* then it is represented by an equation of form:

$F_1(x, y, a) = 0$

(b) If the given family F_2 of curves depends on parameters *a*, *b* (say) then it is represented by an equation of the from:

$F_2(x, y, a, b) = 0$

E. Methods of Solving First Order, First Degree Differential Equations

Differential equations with variable separable

-- If $F(x, y)$ can be expressed as a product $g(x)\,h(y)$, where, $g(x)$ is a function of x and $h(y)$ is a function of y, then the differential equation is said to be of *variable separable equation*. (terms containing y should remain with dy and terms containing x should remain with dx).

$\frac{dy}{dx} = h(y) \cdot g(x)$

Solution: $\int \frac{1}{h(y)}\,dy = \int g(x)\,dx$

Homogeneous differential equations

A differential equation which can be expressed in the form $\frac{dy}{dx} = f(x, y)$ or $\frac{dx}{dy} = g(x, y)$ where, $f(x, y)$ and $g(x, y)$ are homogenous functions of degree zero called *homogeneous differential equation,* e.g. $F_1(x, y) = y^2 + 2xy$.

Solution: we make substitution $\frac{x}{y} = v$ i.e., $x = vy$ and proceed further find general solution by writing $\frac{dx}{dy} = F(x, y) = h\left(\frac{x}{y}\right)$.

Linear differential equations

A differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are constants or functions of x only is called a first order *linear differential equation.*

Solution: Steps solve first order linear differential equation:

(i) Write the given differential equation in the form $\frac{dy}{dx} + Py = Q$ where P, Q are constants or functions of x only.

(ii) Find the Integrating Factor (I.F) $= e^{\int P dx}$.

(iii) Write the solution of the given differential equation as

$y\,(\text{I.F}) = \int (Q_1 \times \text{I.F})\,dy + C$

-- In case, differential equation of the $\frac{dx}{dy} + Px = Q$, then solution is given by:

$x\,(\text{I.F}) = \int (Q_1 \times \text{I.F})\,dx + C$